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Temperature-Time Relations in Canned Foods During Sterilization

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TEMPERATURE-TIME RELATIONS IN CANNED FOODS DURING STERILIZATION.

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Bacteriologists have found that the death rate of the bacteria which cause spoilage in canned goods depends upon the temperature to which the bacteria are subjected. In order, therefore, to establish the bacteriology of canning on a firm scientific basis it is necessary to know the time at which certain critical temperatures are attained by the foods which are subjected to the sterilization process. This paper shows the result of an attempt to work out by a combination of mathematical theory and experiment the temperature-time curves for certain foods in containers of any size and for any practical temperature range.

Apparently very little experimental work has been done along this line, and no attempt has been made, to the author's knowledge, to apply well known mathematical theory. J. Kochs and K. Weinhausen[1] have measured with maximum thermometers the temperatures attained in given times under certain practical conditions, in cabbage, carrots, asparagus, apple musk and green peas. A. W. Bitting and K. G. Bitting[2] have published temperature-time curves for pumpkin, sweet potatoes, tomatoes, peaches, etc., while being heated in a water bath. Aside from the reports of these men very little seems to have been published.

MATHEMATICAL THEORY.

In developing the mathematical thoory it will be assumed that heat penetration is entirely due to conduction. This is obviously not true except in very pasty substances, such as pumpkin, mashed sweet potatoes, etc., but it is thought that in such cases as corn and peas where the convection currents must be mostly very local in character the theory will still apply approximately enough for practical purposes, the convection being in effect equal to an increase of conduction. Also since the center of the can is last to become heated and last to cool off, it is the most important point to be considered and attention will be confined entirely to it.

In order to make the mathematical theory apply without too much complexity, it is necessary to assume rather ideal conditions, namely, that the contents of the can are of uniform temperature throughout before immersion in the sterilizing bath, and that the can is suddenly immersed in a bath which is maintained at a constant temperature.

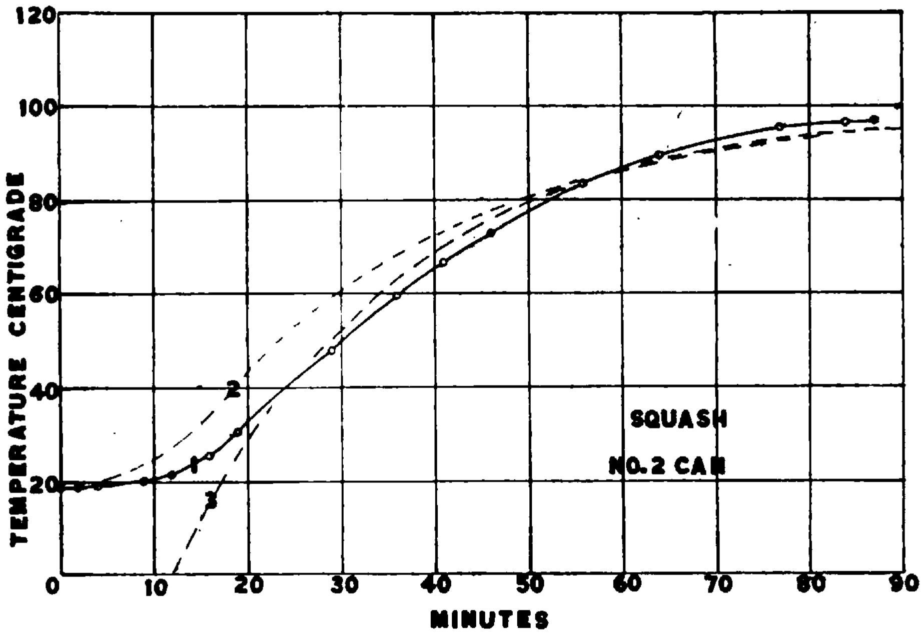

Fig. 1.

The general differential equation of symmetrical heat conduction in a right circular cylinder expressed in cylindrical coordinates is

$$\frac{\partial v}{\partial t} = k \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} \right) \text{-----} (1)$$

v being the variable temperature at any point in the cylinder, r the perpendicular distance of this point from the axis of the cylinder, z the distance from the center of the cylinder along the axis of the base of r, and k the diffusivity $\left(\frac{\text{conductivity}}{\text{density x specific heat}} \right)$ of the material, and t the time[3].

The initial conditions which must be satisfied under the assumed ideal conditions are that

$v = o$ at $r = a$ ....(2)
and $v = o$ at $z = \pm l$................................(3)
and $v = v_0$ at every point of the cylinder when $t = o$........(4)

where a is the radius of the cylinder, 2 l the length, and v_0 the initial temperature of the cylinder.

A general solution of equation (1) satisfying all initial conditions is

$$v=v_o\left[A_l\,J_o\,(\mu_1 r)e^{-k\mu_1^2 t}+\cdots\right]\left[B_l\,\sin\lambda_1(z+l)e^{-k\lambda_1^2 t}\cdots\right] \quad (5)$$

from which

$$v=v_o\left[A_l\,e^{-k\mu_1^2 t}+A_2 e^{-k\mu_2^2 t}+\cdots\right]\left[B_l\,e^{-k\lambda_1^2 t}-B_2 e^{-k\lambda_2^2 t}+\right] \quad (6)$$

when $r=z=o$.

μ is a root of $J_0\,(\mu a)=o$ and λ a root of $\sin 2\lambda l=o$.

EVALUATION OF CONSTANTS.

The roots of $J_0\,(\mu a)=o$ found in the tables[6] are given in the following table. The values of λ which satisfy $\sin 2\lambda 1=o$ are $\lambda n=\frac{n\cdot\pi}{2\,1}$ where n is any positive integer. These are also given below. The values of A are obtained[7] from

$$A_\mu=\frac{-\,\phi}{(\mu_n a)\;J_o(\mu_n a)}$$

and the values of B are $\frac{4}{\pi m}$ where m has all odd positive integral values from 1 to infinity.

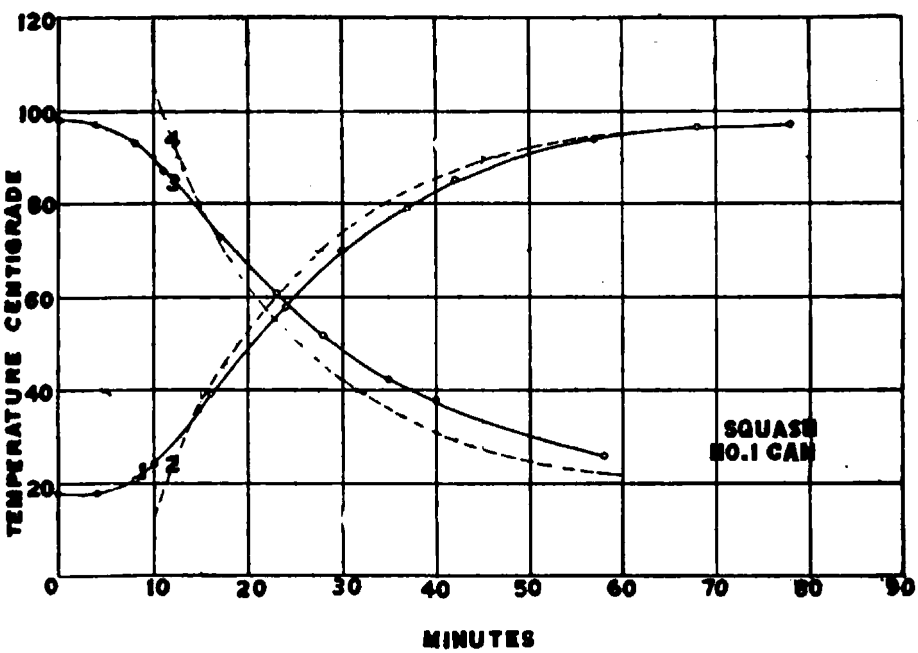

Fig. 2

It will be seen, therefore, that μ depends upon the radius of the cylinder, λ on the length, A and B on the initial conditions.

In table 1 are given the dimensions of commercial tin cans Nos. 1, 2, 2½ and 3, together with the values of μ and λ corresponding. The values of A and B computed from the relations already given are appended.

TABLE I.

		No. 1	No. 2	No. 2½	No. 3
	a	3.35 cm.	4.2 cm.	5.16 cm.	5.2 cm.
μa	1	5.00	5.7	6.03	6.1
2.405	μ_1	.718	.572	.472	.463
5.520	μ_2		1.312		
8.654	μ_3		2.13		
	λ_1	.314	.275	.262	.258
	λ_2		.550		
	λ_3		.825		
	λ_4		1.100		
	λ_5		1.375		
	A_1	1.602	B_1	1.273	
	A_2	—1.065	B_2	.424	
	A_3	.851	B_3	.254	
			B_4	.182	
			B_5	.141	

APPROXIMATE EQUATION.

After considerable time has elapsed all terms after the first in each bracket in equation (6) become negligible and the equation takes the simple form

$$v=v_0 A_1 B_1 e^{-k(\mu_1^2+\lambda_1^2)t} \qquad \text{.................(7)}$$

In the preceding paragraphs the temperature of the bath was considered to be zero so that v_0 represents the initial algebraic temperature difference between the bath and the contents of the can. Expressed in terms of thermometer readings the equation is

$$v^1=v_b + A_1 B_1 v_0 e^{-k(\mu_1^2+\lambda_1^2)t} \qquad \text{.................(8)}$$

v_0 being the negative for heating and positive for cooling. The use of this equation makes unnecessary any shifting of temperature scales. In this equation v^1 represents the variable temperature and v^b the temperature of the bath.

DETERMINATION OF k.

All of the constants except k have been determined; this must be determined experimentally. A sufficiently accurate value of k may be found by using the experimental curve shown in Figure 1 in connection with Equation (8) and the constants given in Table 1. Since sterilization begins at about 60°C., the important part of the curve will be that extending upward from 60°C. We may find the value of k corresponding to the average temperature of this part of the curve which is about 80°C. Figure 1 shows

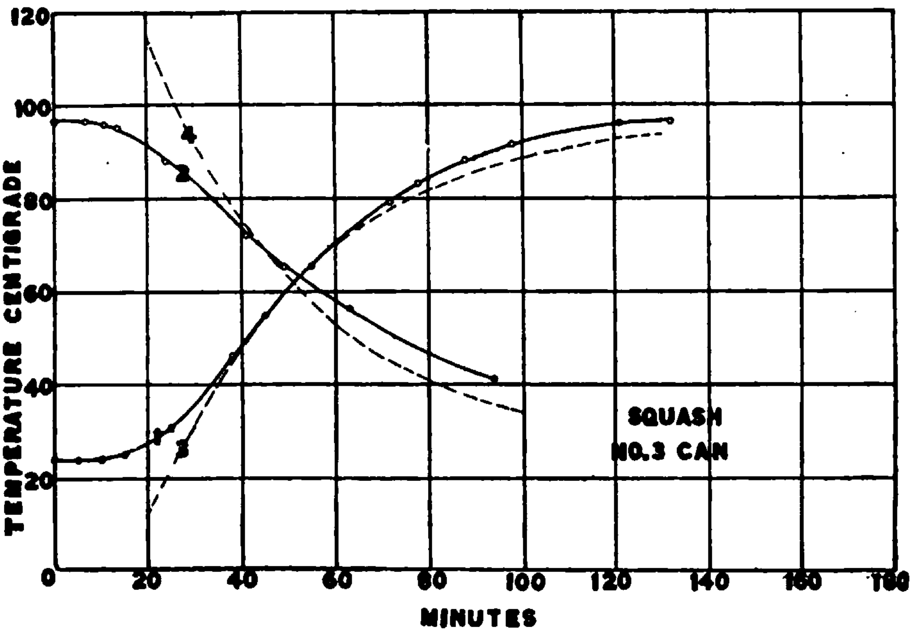

Fig. 3.

that 52 minutes were required for this temperature to be attained. Making appropriate substitutions in Equation (8) we have

$$80 = 98 - 1.602 \times 1.272 \times 8.05 e^{-k(\overline{.572}^2 + \overline{.275}^2)52} \qquad \text{.........(9)}$$

from which k = .105.

GLASS CANS.

Extensive experimentation has not been done with glass cans but it seems that the effect of the glass is negligible, and that the rate of heat flow is about the same as in tin. From theoretical considerations this is to be expected because glass is at least as good a conductor as the contents of the can in the cases here cited, and being very thin as compared to the radius of the can very little effect is to be expected.

DISCUSSION OF CURVES.

Figure 1, Curve 1 shows the experimental heating curve for central point of a No. 2 can of squash. Curve 2 is the curve obtained for a No. 2 can by placing the appropriate constants in equation (6). Curve 3 shows the results obtained by using equation (8).

Figure 2 shows curves for a No. 1 can of squash, the theoretical curve being computed by use of the value of k previously found from the experimental curve of the No. 2 can.

Figure 3 shows similar curves for a No. 3 can.

Figure 4 shows a set of curves for squash cooked in a steam bath at 129°C. The value of k(=.105) previously used for the water bath was used here in computing the theoretical curves.

Similar results were obtained for corn.

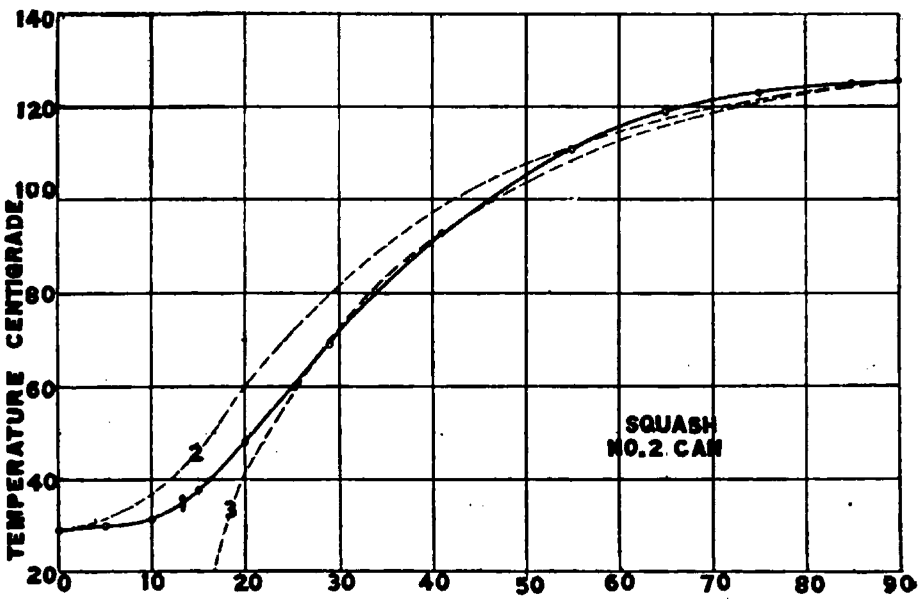

Fig. 4.

It is evident, therefore, that if the time required for a can of given dimensions to acquire a certain temperature be given, it is possible to construct a complete temperature-time curve for a can of any size and for any practical temperature range with a fair degree of accuracy.

ARRANGEMENT OF APPARATUS.

The hot water sterilizing bath consisted of a large open vessel about 32 cm. in diameter, and 31 cm. high filled with tap water.

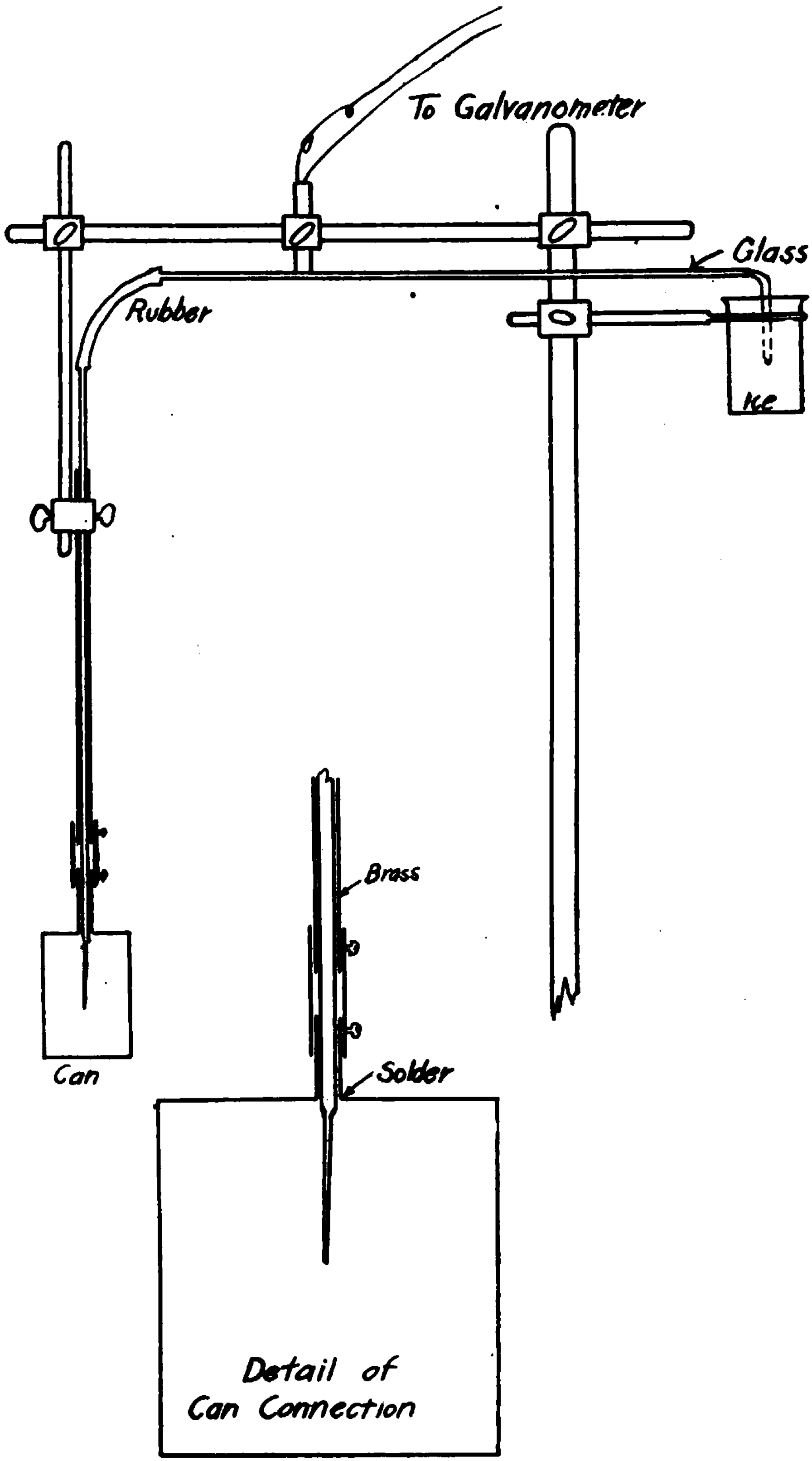
To Galvanometer
Glass
Rubber
Ice
Can
Brass
Solder
Detail of
Can Connection

Fig. 5.

The water was heated by a number of Bunsen flames. After the water had begun to boil the can was lowered suddenly into it, and the temperature readings made at frequent intervals.

The temperature readings were made with a thermocouple and sensitive galvanometer (1.88 x 10^{-6} volt/mm.) The thermocouple was of constantan (No. 27) and copper (No. 30) mounted as shown in Fig. 5. The wires were protected by enclosure in glass and rubber tubing. The glass tube was narrowed down to about 2 mm. diameter at the point where it enters the can. The can was supported by a brass tube held fast by solder. In order to make the interchange of cans easy the brass tube was made in two pieces and held together by a collar. The wires of the thermo-junction were allowed to project about 2 mm. beyond the end of the glass tube and were sealed in shellac. This did not form a permanent seal and was renewed for each set of readings. The whole outfit was mounted as compactly as possible by use of laboratory clamps and rods to enable the can to be lowered readily into the bath.

The temperature readings made with this arrangement are accurate to within about 1°C. For calibration purposes a mercury thermometer was used as a standard.

When the steam bath was used it was necessary to lead the thermocouple wires in through a steam tight joint capable of withstanding a pressure of twenty pounds per square inch gauge. This was accomplished by inserting the couple in a thin glass tube closed at one end, and held in place by a rubber gasket as shown in Figure 6; the closed end which contains and protects the junction being placed at the center of the can. The presence of the thin glass surrounding the junction did not prevent the junction from attaining the temperature of the surrounding material promptly. A test showed that in about one-half minute after immersion in boiling water the junction was practically at the temperature of the water.

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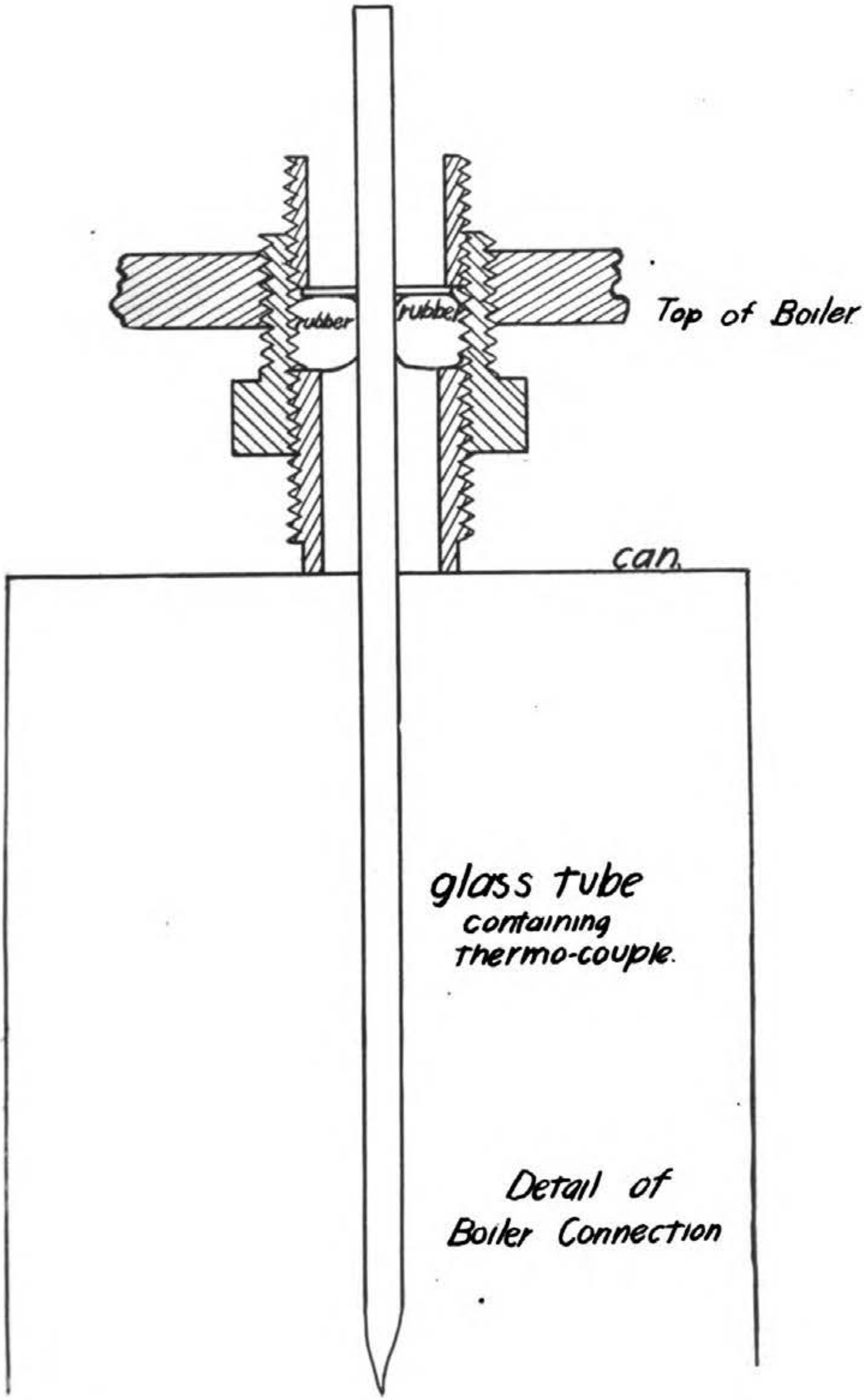

rubber
rubber
Top of Boiler
can
glass tube
containing
thermo-couple.
Detail of
Boiler Connection

Fig. 6

Heat, pp. 542-3, Macmillan & Co., 1894. Ingersoll & Zobel, Mathematical Theory of Heat Conduction, pp. 9-14. Ginn & Co.

4. $$J_0\,(\mu r) = 1 - \frac{(\mu r)^2}{2^2} + \frac{(\mu r)^4}{2^2 \cdot 4^2} - \frac{(\mu r)^6}{2^2 \cdot 4^2 \cdot 6^2} + $$

See Byerly, an Elementary Treatise on Fourier's Series, etc., p. 219.

5. Byerly, l. c., p. 286.

6. Byerly, Harmonic Functions, p. 218. (Edited by Merriam & Woodward.) Published by Wiley & Sons.

Byerly, Introduction to the Theory of Fourier's Series, etc., p. 39.

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